Verified Construction of Static Single Assignment Form

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Implementation Complexity of Construction Algorithms

- Dominance frontier-based algorithms
  - Introduced in *An Efficient Method of Computing SSA Form* [Cytron et al., TOPLAS '91]
  - Used by GCC, LLVM, . . .
  - High implementation complexity
  - No existing formal verification

- Algorithms designed for simplicity
  - *Simple Generation of SSA Form* [Aycock and Horspool, CC '00]
  - Two-step algorithm:
    1. "Really Crude" phase: maximal SSA form
    2. Minimization phase

SSA Construction in Verified Compilers

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- Vellvm [Zhao et al., PLDI '13]
  - Formalization of the LLVM IR
  - Uses Aycock and Horspool's algorithm
    - Proof of semantic correctness
    - No proof of minimality
- CompCertSSA [Barthe et al., PLDI '13]
  - Extends the verified CompCert C compiler with an SSA midend
  - *Translation Validation* approach:
    - Untrusted implementation of Cytron et al.'s algorithm
    - Verified validator
    - No proof/validation of minimality

Construction Algorithm by Braun et al.

Simple and Efficient Construction of Static Single Assignment Form [Braun et al., CC ’13]

Simplicity

- Does not use dominance frontiers or any other analyses

Efficiency

- Shown to be on par with LLVM’s construction pass
- Used in libfirm and the Go compiler

Output size

- Pruned for all inputs
- Minimal for reducible/all inputs

Formalization

A functional variant of Braun et al.'s core algorithm in Isabelle/HOL

- CFG-based transformation
- Minimal only for reducible inputs

Algorithm split into basic parts:

1. Pruned SSA form
2. Minimization

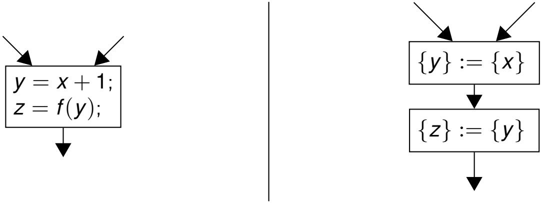
Goal

- Complete verification
- Special focus on quality guarantees

Formalization – CFG Abstraction

Abstract, minimal CFG representation:

- Graph structure
- Defs and uses per basic block
- Assumption: definite assignment
- Assumption: no intra-block data dependencies



Formalization – SSA Definition

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Definition (SSA CFG)

A CFG with ϕ functions is an *SSA CFG* if

- every SSA value is defined at most once
- all ϕ functions are well-formed: #arguments = #CFG predecessors
- definite assignment also holds for all ϕ functions (*strict* SSA form)
- it is in *conventional* SSA form (for Cytron et al.'s minimality definition)

Definition (Valid SSA translation)

An SSA CFG is a *valid SSA translation* of a CFG if

- it only adds ϕ functions and renames variables
- ϕ functions only reference SSA values of the same variable

Proof of Correctness

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Theorem (Semantics Preservation)

If G' is a valid SSA translation of G, then G and G' are semantically equivalent.

Formalization – Pruned Construction

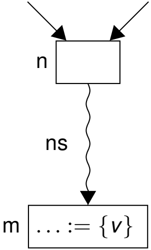
Definition (Prunedness)

An SSA CFG is in *pruned* SSA form if all ϕ functions are live.

- Cytron et al.: iterate dominance frontiers of def sites, use liveness analysis for prunedness
- Braun et al.: backwards search from use sites, implicitly pruned

lemma *phiDefNodes v* $= \{n.$
length (*predecessors n*) $> 1 \land$ — *n* is a join point
$\exists$ *ns m*. *n*–*ns*→*m* $\land$
v $\in$ *uses m* $\land$
$\forall$ *n* $\in$ *ns*. *v* $\notin$ *defs n*
$\}$

(the last three conditions: *v* is live at *n*)



Formalization – Minimization

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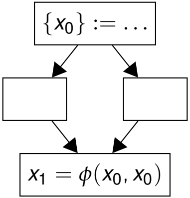
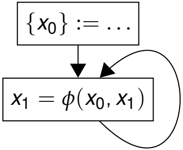
Aycock and Horspool: for reducible inputs, sufficient to remove all *trivial* ϕ functions





Implementation

Define a graph transformation that removes a single trivial ϕ function, then close over it via a fixed-point iteration.

Proof of Minimality

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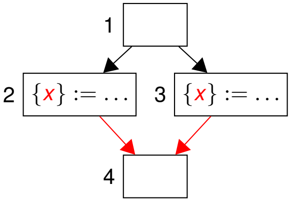
Definition (Convergence property)

There is a ϕ function wherever paths from two definitions of a variable converge.

Definition (Minimality [Cytron et al.])

An SSA CFG is in *minimal* SSA form if it *only* contains ϕ functions satisfying the convergence property.



Theorem (Trivial ϕ criterion)

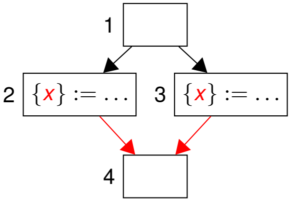
$\textit{reducible } g \wedge \neg \textit{hasTrivPhis } g \Longrightarrow \textit{cytronMinimal } g$

Isabelle proof (~1000 LoC) closely follows the handwritten proof by Braun et al. (~1.5 pages)

Proof of Minimality

A single major modification was needed:

- The handwritten proof uses the convergence property, which does not necessarily hold after pruning
- Corrected version: It is necessary to insert ϕ functions where paths from definitions of a variable converge and the variable is live



This leads to an even stronger minimality theorem:

Theorem (ϕ-count minimality)

A translated SSA CFG in both minimal and pruned SSA form has the minimum number of ϕ functions among all valid translations.

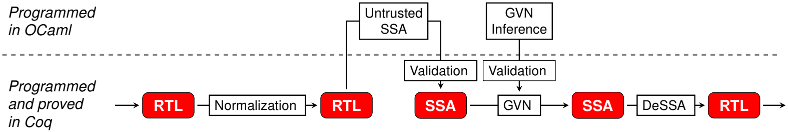
Verification Results

We proved that our formalization of Braun et al.'s algorithm computes

- ✓ an SSA CFG
- ✓ a valid translation of the input CFG
  - $\Rightarrow$ Semantic equivalence
- ✓ pruned SSA form
- ✓ minimal SSA form for reducible input CFGs

CompCertSSA Integration

Barthe et al. [PLDI '13]



We replaced the construction + validation with an OCaml extraction of our verified Isabelle code

- Refined implementation to optimize asymptotics
- Some unverified OCaml glue code needed for interoperability

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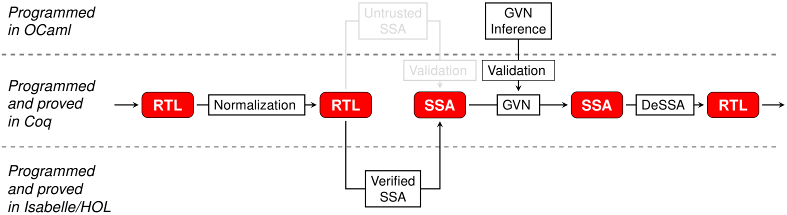

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CompCertSSA Integration – Performance

Benchmark	Our formalization				
	Pruned	Minimization	Glue	Total	#ϕ
177.mesa	0.46 s	0.64 s	0.20 s	1.31 s	4884
186.crafty	0.16 s	0.16 s	0.15 s	0.47 s	1169
300.twolf	0.26 s	0.40 s	0.10 s	0.76 s	2259
spass	0.79 s	1.08 s	0.53 s	2.41 s	15192

Benchmark	CompCertSSA				
	LV Analysis	ϕ Placement	Validation	Total	#ϕ
177.mesa	0.66 s	0.33 s	0.17 s	1.16 s	4884
186.crafty	0.28 s	0.30 s	0.27 s	0.84 s	1169
300.twolf	0.42 s	0.24 s	0.16 s	0.82 s	2259
spass	1.38 s	1.16 s	0.65 s	3.20 s	15168

Runtime on an Intel Core i7-3770 with 3.40 GHz and 16 GB RAM.

Conclusion

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Our functional implementation of Braun et al.'s algorithm is

- simple enough for a complete verification in Isabelle/HOL
- efficient for real-world inputs: on par with CompCertSSA's construction pass

We further formally proved that

- Aycock and Horspool's trivial ϕ criterion is correct
- minimality and prunedness together imply a minimum number of ϕ functions

Complete formalization available at
`http://pp.ipd.kit.edu/ssa_construction`